Maulana Azad College

Department of Mathematics

MathZiN

Volume--1

2023-24

INDEX

Principal's Message

With great delight, I extend my warmest greetings to all members of our esteemed institution as we present the latest edition of our Mathematics Magazine. Witnessing the dedication and enthusiasm of our students and faculty in curating this publication fills me with immense pride.

Mathematics is more than just a subject; it is a universal language that reveals the mysteries of the universe and equips us with problem-solving skills that transcend academic boundaries. In this edition, you will discover a diverse range of articles, problems, and insights that highlight the multifaceted nature of mathematics and its applications across various fields.

I extend my heartfelt gratitude to the editorial team, comprised of both students and faculty members, for their tireless efforts in bringing this magazine to life. Their unwavering commitment to promoting mathematical knowledge and fostering a love for the subject within our community is truly commendable.

May this magazine serve as a source of inspiration, igniting curiosity and a passion for the endless possibilities within the realm of mathematics. I hope it not only deepens our understanding of the subject but also strengthens the bonds among us, fostering a sense of camaraderie.

Let us celebrate the achievements of our students and faculty, and continue to nurture a culture of learning and excellence in our institution. Thank you for your continued support, and I look forward to witnessing the ongoing growth and success of our mathematics community.

Principal
Maulana Azad College

Message from the Head of the Department
Dr. Somnath Bandyopadhyay,
Maulana Azad College, Kolkata

It is with immense pleasure that I extend a warm welcome to all of you to the vibrant realm of our College Mathematics Magazine. As the Head of the Mathematics Department, I am thrilled to witness the launch of this remarkable initiative that celebrates the beauty, diversity, and intellectual richness of the mathematical world.

Our College Mathematics Magazine is not just a publication; it is a testament to the passion, curiosity, and brilliance that define our mathematics community. Mathematics, as we know, is not merely a subject but a universal language that unveils the secrets of the universe. Through this magazine, we aim to cultivate a deeper appreciation for the elegance and transformative power that mathematics brings to our lives.

Within these pages, you will find a diverse collection of articles, features, and insights that highlight the extraordinary achievements of our students and faculty. This magazine serves as a platform to showcase the exceptional talent and creativity that thrive within our department.

I encourage each of you—whether a seasoned mathematician or someone just embarking on their mathematical journey—to actively engage with the content. This magazine is designed as a space for everyone to learn, explore, and be inspired by the boundless possibilities of mathematics.

I would like to extend my heartfelt gratitude to the dedicated team of students and faculty members whose hard work and enthusiasm have brought this publication to life. Your unwavering commitment to excellence is evident on every page, and I am confident that this magazine will be a source of pride and inspiration for our mathematics community.

As we embark on this exciting journey together, let us celebrate the beauty of mathematics and the intellectual curiosity that drives us to explore its infinite depths. May this magazine be a source of inspiration, motivation, and a shining example of the enduring spirit of the Mathematics Department at Maulana Azad College.

Wishing you all a fantastic experience as you delve into the pages of our inaugural Mathematics Magazine!

Head
Mathematics Department,
Maulana Azad College

TEACHER'S EDITORIAL

DR. NANDA DAS

DR.BABLI SAHA

Dear Readers,

It is with great pleasure and enthusiasm that we welcome you to the latest issue of Mathematics Magazine. As we embark on this mathematical journey together, we are reminded of the profound beauty and significance that mathematics holds in our lives.

In this edition, we have curated a diverse collection of articles that traverse the expansive landscape of mathematics. From Algebra, Number Theory, and Real Analysis in pure mathematics to Mathematical Biology and Applied Physics, these contributions reflect the richness of the field. Our contributors—ranging from current and former students to esteemed members of our faculty—have crafted insightful pieces that we are confident will engage, inspire, and perhaps challenge your understanding of the mathematical universe.

Beyond the articles, this issue highlights various programs and extracurricular activities organized by the department. These initiatives showcase not only the depth of mathematical knowledge but also its profound impact on the world around us.

We extend our sincere gratitude to all contributors for sharing their expertise and passion for mathematics, and to our dedicated editorial team for their tireless efforts in bringing this issue to life.

We hope that Mathematics Magazine continues to serve as a source of intellectual stimulation and a catalyst for fostering a deeper appreciation of the mathematical sciences.

Happy reading!

Nanda Das

Associate Professor,
Mathematics Department,
Maulana Azad College

Babli Saha

Associate Professor,
Mathematics Department,
Maulana Azad College

STUDENT'S EDITORIAL

-A Journey Beyond Numbers

Dear Readers,

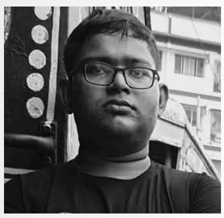
Welcome to this latest installment of our college Mathematics Magazine! As we venture into the vast expanse of mathematical marvels, we are reminded of the sophistication, accuracy, and breathtaking beauty that mathematics has to offer. Often regarded as the most fundamental of all fields, mathematics serves as the cornerstone for subjects spanning from Physics and Chemistry to social sciences like Economics and cutting-edge fields such as Computing and Data Science. In this issue, we embark on a journey beyond mere numerals, exploring the intricacies of mathematical concepts that not only shape our understanding of the world but also ignite our imagination.

This magazine is the culmination of the tireless efforts and dedication of the students from the Mathematics Department, who have been expertly guided and supported by our esteemed faculty members. Our objective through this publication is to demonstrate that mathematics is far more than an abstract notion confined to textbooks and classrooms; it is a dynamic and powerful tool with countless practical applications. We, the student editors of MATHZIN, extend our sincerest appreciation to all the students and professors of our department for their invaluable contributions. We hope you find this edition filled with captivating insights and astounding facts. May the beauty of mathematics continue to inspire and enthrall us all!

Best Regards,

Saswati Ghosh

Saanway Dutta



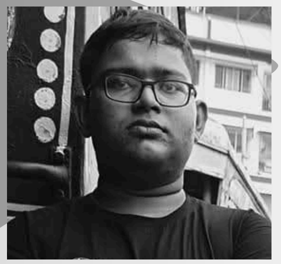
Rohan Mondal

STUDENT'S CORNER

e-volution

-Saanway Dutta

রামানুজনের জীবনে গণিতের অবদান

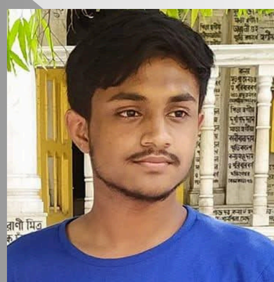


-রোহন মণ্ডল

History of Limits

-Nandini Bose

An Introduction to Bakhshali Manuscript



-Soughata Deb

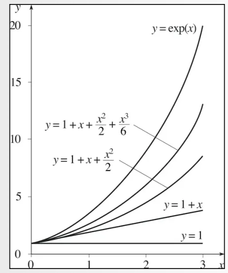
Inca's Quipu: A Historical Wonder

-Saswati Ghosh

e-volution

-Saanway Dutta

The first appearance of e in mathematics was in the appendix to an edition of Jhon Napier's book on logarithms,published in 1618 .Where e was used as the base of logarithm which did not clarification to e.Later Bernouli discovered fundamental properties of e while studying compound interest .He noticed that the sequence approached a limit as the compounding intervals became smaller
In probing the properties contineous compound interset he had a limit



$$\lim_{n\to\infty}(1+\frac{1}{n})^n$$

he also noticed that the limit of the above sequence lies between 2 and 3 .

consider the under mentioned problem,
imagine a bank account has an initial deposit of RS 1.00 total paying 100 percent in interest annually. The value of the account will be valued at RS 2.00 at the end of the year after the first interest has been credited. The question in Bernoulli's mind was what would happen if the interest given by the bank had been credited more frequently throughout the following financial year?
Bernoulli calculated that if the interest had been credited twice in any specific year, the interest rate 50 percent for a time period of 6 months. As a result, the RS 1 in the opening of the account is multiplied by 1.5 twice in a year, giving the bank

$$1.00 \times (1+\tfrac{1}{2})^2 = 2.25$$

If a similar situation was taken but now with the interest being compounded quarterly, it would yield

$$1.00 \times (1+\tfrac{1}{4})^4 = 2.4114..$$

Taking a scenario where the interest was compounded monthly would yield

$$1.00 \times (1+\tfrac{1}{12})^{12} = 2.613035..$$

If n compounding time frames and intervals are taken, the interest for each interval can be calculated by dividing 100 percent by n time frames which yields at the end of a financial year a total account valued at $(1+\frac{1}{n})$ raised to n, $1.00\times(1+\frac{1}{n})^n$

Later Leonhard Euler denote this constant as e. In 1784 Euler published introduction in *Analysin infinitorum* in which he calculated e to 18 decimal places such that

$$2.71828182845904523536028747135 27$$

He showed that e was the limit in Bernouli's compound interset problem and that e could also be calculated from the infinte series given bellow

$$e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}........$$

The generalized exponential function e^x

$$e = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}........$$

In Euler identity ,

$$e^{ix} = \cos x + i \sin x$$

i is the square root of -1 .When $x = \pi or 2\pi$,the formula yields an elegant expressions relating π, e, i

$$e^{i\pi} + 1 = 0$$

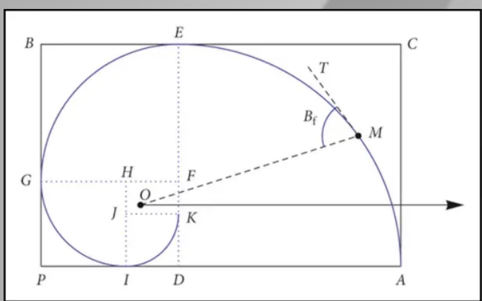
Natur'e'

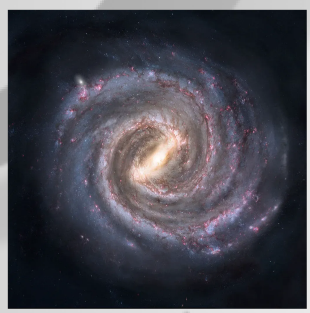
Another interesting property that e has is that if we put it in the polar coordinate system, we will observe a figure known as 'the Logarithmic Spiral
The Natural number in nature Patterns such as the logarithmic spiral can easily be observed in nature, from the curvature of flowers and seashells to the spin of tornadoes and galaxies!

The equation of the equiangular spiral in polar cordinates is

$$r = ae^{k\theta}$$

where a and k are constants and $a > 0$





e **day**
e-Day celebrates the mathematical constant e, which has a value of 2.7182818. It is celebrated on February 7 in countries that follow the month/day (m/d) date format. This is because the first two digits of the date $-2/7-$ correspond to the first two digits of the constant.

রামানুজনের জীবনে গণিতের অবদান -রোহন মণ্ডল

আমাদের চারপাশে বা ইতিহাসের পাতায় কখনো এমন কিছু বিরল ব্যক্তিত্বের সন্ধান পাওয়া যায় যে তাদের নিয়ে আলোচনা করার সময় একটি বিষয় বা দিক আমাদের চোখের সামনে ভেসে ওঠে। যাকে বাদ দিয়ে ব্যক্তির আলোচনা কেবল অর্থহীন নয়, অসম্ভব। এর সবথেকে ভালো উদাহরণ বোধহয় ব্যক্তি রামানুজন ও বিষয় গণিত।

রামানুজনের ব্যক্তিগত সান্নিধ্য বা তার অগণিত গণিত ভাবনার আভাস যারা পেয়েছেন তাদের প্রত্যেকে রামানুজনের অনন্যসাধারণ প্রতিভার দ্যুতি, অসাধারণ গণিত মানসিকতা, অদ্ভুত বিশ্লেষণী ক্ষমতা ও বিস্ময়কর মেধার ব্যাপ্তি তে সম্মোহিত হয়েছেন, হারিয়ে গেছেন অনুভূতির গভীরে।

গণিতে রামানুজনের অবদান আজ সারাবিশ্বে স্বীকৃত। কিন্তু রামানুজনের জীবনে গণিতের অবদান? গণিতের সাধনা না করলে কেমন হতো দক্ষিণ ভারতের এক ক্ষুদ্র শহরের সামান্য এক কাপড়ের দোকানের কর্মচারীর জ্যেষ্ঠ পুত্রের জীবন? এবং কেমন করে গণিতই তার জীবনের ধ্রুবতারা হয়ে উঠল- আজ বরং সেই বিষয়েই আলোচনা করা যাক।

শিশু বয়স থেকেই রামানুজনের অসাধারণ স্মৃতিশক্তি ছিল। তিনি মায়ের কাছে ভক্তিগীতি, স্তোত্রপাঠ ও পুরানের বহু কাহিনী মুখে মুখে শুনেই শিখে ফেলেছিলেন। তার সংস্কৃত উচ্চারণ ছিল শুদ্ধ। মা ও অন্যান্য প্রতিবেশীদের রামায়ণ, মহাভারত,বেদ ও উপনিষদের শ্লোক আকৃতি করে শোনাতেন বালক রামানুজন। অর্থাৎ তৎকালীন যুগে একজন প্রকৃত ব্রাহ্মণ হয়ে ওঠার সমস্ত প্রথাকরণ, রীতিনীতি খুব নিবিড়ভাবে আয়ত্ত করেছিলেন তিনি। সাথে সাথেই নিবিড় হচ্ছিল গণিতের সাথে তাঁর আত্মার বন্ধন।

কিশোর বয়সে যাত্রা ও নাটকের প্রতি তার কৌতূহল কম ছিলো না। রাতের অন্ধকারে পাঁচ-ছয় মাইল রাস্তা পেরিয়ে তিনি সেসব দেখতে যেতেন। আবার ধর্মীয় অনুষ্ঠানেও তার সমান আগ্রহ। মাঝে মাঝে মন্দিরে গিয়ে স্তোত্র পাঠের মাধ্যমে পূজা করতেন তিনি। তবে পরবর্তী তে এমন ও হয়েছে যে রামানুজন গণিতচর্চায় এতটাই নিমগ্ন যে মন্দিরে যাওয়ার সময় পর্যন্ত পেতেন না।

ছেলেবেলা থেকেই তিনি ছিলেন ভাবুক ও চিন্তাশীল। তার উদ্ভট সব প্রশ্নে মাস্টারমশাইরা যতো না বিস্মিত হতেন, তার চেয়ে বেশি অবাক হতেন সহপাঠীরা। তারা যেসব অঙ্ক পারতো না তা রামানুজনকে সমাধান করতে দিতেন। আর তিনিও চারপাশ ভুলে গণিত নিমগ্নতায় বিভোর হয়ে থাকতেন। তাই একথা বললে অত্যুক্তি হয়না যে শৈশবেই আপাত বন্ধুহীন নিঃসঙ্গ রামানুজনের কাছে গণিত ই ছিলো একমাত্র সঙ্গী, তাঁর কৌতূহলী মনের পরম আশ্রয়স্থল।

রামানুজন যখন টাউন হাইস্কুলের ছাত্র তখন কলেজে পড়া দুজন ছাত্রকে তাদের বাড়িতে বোর্ডার হিসেবে রাখা হয়। গণিতের প্রতি তাঁর তীব্র আকর্ষণ দেখে তারা তাদের জানা গণিত রামানুজনকে শেখাতে শুরু করেন। কিন্তু মাত্র কয়েকমাসের মধ্যে তাদের অংকের যাবতীয় জ্ঞানভান্ডার উজাড় হয়ে যাওয়ার পর রামানুজনের পীড়াপীড়িতে কলেজ লাইব্রেরি থেকে এস. এল. লোনীর ত্রিকোণমিতি বই নিয়ে এসে দেন তাকে। মাত্র তেরো বছর বয়সে তা একক প্রচেষ্টায় সমাধান করেন রামানুজন। ভাবলে বিস্মিত হতে হয় যে কতখানি ভালোবাসা ও নিষ্ঠা থাকলে গণিত এভাবে কারোর কাছে নিজেকে উজাড় করে দেয়।

তবে নিঃসন্দেহে তাঁর গণিত চর্চার turning point ছিলো ১৯০৩ সাল। যখন G. S. Carr এর লেখা 'A Synopsis of Elementary Results in Pure and Applied Mathematics' বইটি তার হাতে আসে। এতে ছিলো বীজগণিত, জ্যামিতি, ত্রিকোণমিতি, অবকল, সমাকল সমীকরণ প্রভৃতি শাখা নিয়ে নানা সূত্র ও উপপাদ্য। যা রামানুজনের বুভুক্ষু মনের ক্ষুধা আপাত ভাবে মেটাতে সক্ষম ই হয়নি, পরবর্তীতে তার জীবনের গতিপ্রকৃতি অনেকটাই নিশ্চিত করে দিয়ে যায়। রামানুজনের কাছে বইটির প্রতিটি উপপাদ্য যেনো তাঁর নিজস্ব গবেষণা-প্রকল্প। তিনি একের পর এক সূত্র ও উপপাদ্যের আবিষ্কারের আনন্দে মাতলেন। ১৯০৩ সালের পর থেকে তিনি এগুলি লিখে রাখতে শুরু করলেন। যা পরবর্তীতে তাঁর বিখ্যাত নোটবই রূপ পরিগ্রহন করে।

১৯১০ সালের শুরুর দিকে চাকরির আশায় রামানুজন যখন তিরুকৌলুলের ডেপুটি কালেক্টর ভি রামস্বামী আইয়ারের সঙ্গে দেখা করতে যান তখন তাঁর সম্বল বলতে শুধুই গণিতে লব্ধ জ্ঞান আর তাঁর যাবতীয় কাজকর্মের দলিল সেই নোটবই।

গণিত আর এই নোটবই এর তাঁকে বেকারত্বের জ্বালা থেকে মুক্তি দিয়েছিল তাই নয় কালাপানি পেরিয়ে তাঁর বিদেশ যাত্রার ও বিশ্বগণিতের আঙিনায় এক আপাততুচ্ছ ভারতীয়ের গৌরবজ্জ্বল কীর্তি স্থাপনের ভিত্তিপ্রস্তর ও স্থাপন করে দিয়েছিল। ভারতীয় গণিতবিদের সাফল্যের কাহিনী আমরা কমবেশি সবাই জানি। তাই সে প্রসঙ্গ এখন থাক।

কিন্তু কোথাও গিয়ে মনে প্রশ্ন জাগে এই যে গণিতে রামানুজনের বিশ্বজোড়া এতো খ্যাতি, এতো জয়জয়কার, তিনি কি নিজেও এসব চেয়েছিলেন?

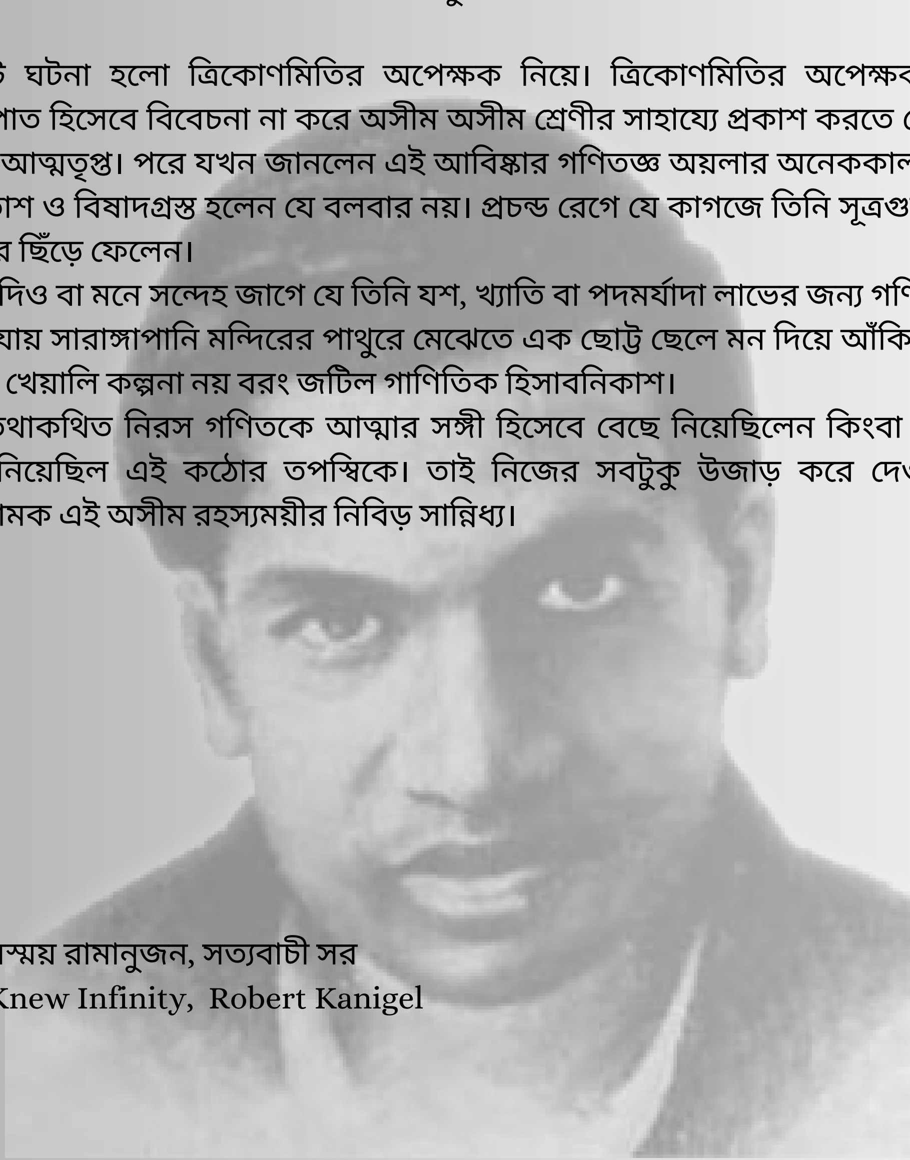
এর উত্তর খোঁজার আগে দুটো ছোট্ট ঘটনার কথা জেনে নিই বরং।

১৮৯৭ সালের প্রাথমিক পরীক্ষায় রামানুজন পাটিগণিতে ৪৫ এর মধ্যে ৪২ নম্বর পেলেন। অথচ তাঁর এক সহপাঠী পেলেন ৪৩ নম্বর। যুগপৎ বিস্মিত ও ক্ষুব্ধ রামানুজন সহপাঠীর সাথে কথা পর্যন্ত বললেন না। এবং সেই সহপাঠী যখন তাকে সান্ত্বনা দিতে এলেন এই বলে যে বাকি বিষয়ে রামানুজন তার থেকে বেশি পেয়েছে তখন রামানুজন প্রায় কেঁদে ফেলেছিলেন।

এমনতরো আরকেটি ঘটনা হলো ত্রিকোণমিতির অপেক্ষক নিয়ে। ত্রিকোণমিতির অপেক্ষক গুলিকে সমকোনী ত্রিভুজের বাহুর অনুপাত হিসেবে বিবেচনা না করে অসীম অসীম শ্রেণীর সাহায্যে প্রকাশ করতে পেরে রামানুজন তখন আবিষ্কারের আনন্দে আত্মতৃপ্ত। পরে যখন জানলেন এই আবিষ্কার গণিতজ্ঞ অয়লার অনেককাল আগেই করে গেছেন তখন তিনি এতটা হতাশ ও বিষাদগ্রস্ত হলেন যে বলবার নয়। প্রচন্ড রেগে যে কাগজে তিনি সূত্রগুলি লিখে রেখেছিলেন টা টুকরো টুকরো করে ছিঁড়ে ফেলেন।

উপরিউক্ত ঘটনায় যদিও বা মনে সন্দেহ জাগে যে তিনি যশ, খ্যাতি বা পদমর্যাদা লাভের জন্য গণিত চর্চায় এসেছিলেন, পরক্ষনেই মনে পড়ে যায় সারাঙ্গাপানি মন্দিরের পাথুরে মেঝেতে এক ছোট্ট ছেলে মন দিয়ে আঁকিবুকি করে চলেছে। না তা শিশুমনের কোনো খেয়ালি কল্পনা নয় বরং জটিল গাণিতিক হিসাবনিকাশ।

আসলে রামানুজন তথাকথিত নিরস গণিতকে আত্মার সঙ্গী হিসেবে বেছে নিয়েছিলেন কিংবা বোধহয় গণিতই তাঁর শিশু বয়সেই চিনে নিয়েছিল এই কঠোর তপস্বিকে। তাই নিজের সবটুকু উজাড় করে দেওয়ার বিনিময়ে তিনি পেয়েছিলেন গণিত নামক এই অসীম রহস্যময়ীর নিবিড় সান্নিধ্য।



তথ্য সূত্র :

১/ গণিত জগতের বিস্ময় রামানুজন, সত্যবাচী সর

২/ The Man Who Knew Infinity, Robert Kanigel

SRINIVASA RAMANUJAN

RAMANUJAN'S MANUSCRIPT

History of Limits

-Nandini Bose

The concept of limits has a rich and fascinating history that spans thousands of years. Let's take a journey through time and explore the key milestones in the development of limits:

Ancient Civilizations (500 BCE - 500 CE)

1. Zeno's Paradoxes (500 BCE): The ancient Greek philosopher Zeno posed paradoxes that involved infinite sequences and limits, laying the groundwork for future mathematicians.
2. Archimedes (287-212 BCE): Archimedes developed the method of exhaustion, a precursor to integration, which relied on limits to calculate areas and perimeters.

Medieval Period (500-1500 CE)

1. Indian Mathematicians (500-1200 CE): Mathematicians like Aryabhata, Brahmagupta, and Bhaskara made significant contributions to the concept of limits, including the use of infinite series.
2. Ibn Sina (Avicenna) (980-1037 CE): The Persian polymath Ibn Sina wrote about the concept of limits in his book "The Book of Healing."

Renaissance and Enlightenment (1500-1800 CE)

1. Bonaventura Cavalieri (1598-1647 CE): The Italian mathematician Cavalieri developed the method of indivisibles, which used limits to calculate volumes and areas.
2. Johannes Kepler (1571-1630 CE): Kepler used limits to calculate the areas and perimeters of shapes, laying the foundation for modern calculus.
3. Sir Isaac Newton (1642-1727 CE) and Gottfried Wilhelm Leibniz (1646-1716 CE): Both Newton and Leibniz developed the concept of limits as part of their work on calculus.

Modern Era (1800-present)

1. Augustin-Louis Cauchy (1789-1857 CE): Cauchy formalized the concept of limits using epsilon-delta notation, providing a rigorous foundation for calculus.
2. Karl Weierstrass (1815-1897 CE): Weierstrass further developed the theory of limits, introducing the concept of uniform convergence.
3. Henri Lebesgue (1875-1941 CE): Lebesgue's work on measure theory and integration relied heavily on the concept of limits.

The concept of limits has evolved over thousands of years, with contributions from mathematicians and philosophers across various cultures and civilizations. Today, limits remain a fundamental concept in mathematics, underpinning calculus, analysis, and many other areas of mathematics.

An Introduction to Bakhshali Manuscript

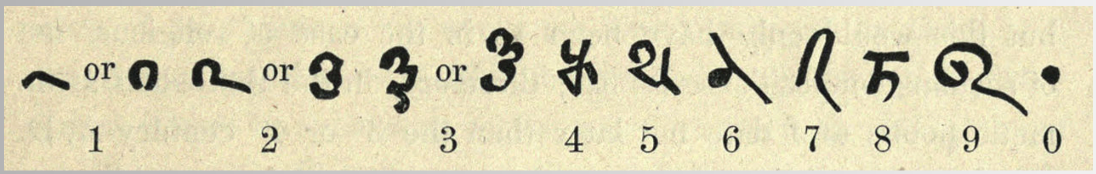
-Soughata Deb

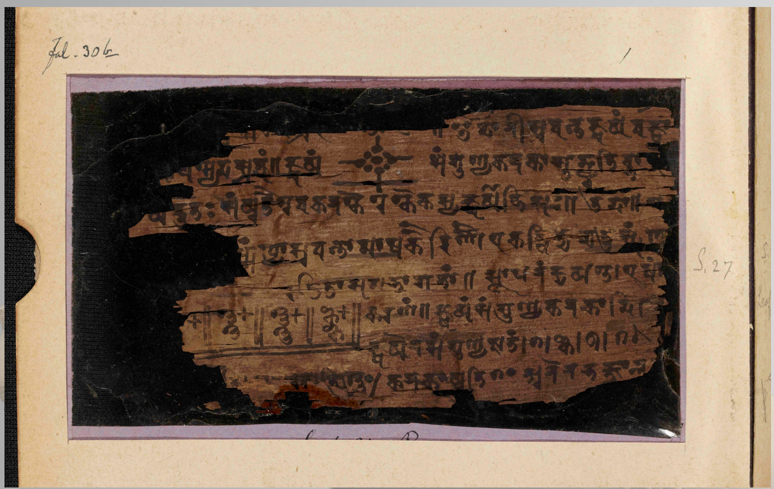
The Bakhshali manuscript, an antiquated Indian mathematical text inscribed on over 70 birch bark folios, is remarkable for featuring a dot symbolizing the concept of 'zero'. The manuscript's age has long fascinated scholars, with many assuming it originated in the 9th century CE. However, researchers from the University of Oxford and the Bodleian Libraries have conducted radiocarbon dating, revealing that the manuscript actually dates back to between the 3rd and 4th centuries CE. This discovery establishes the Bakhshali manuscript as the earliest known recorded source of the zero symbol used universally today.

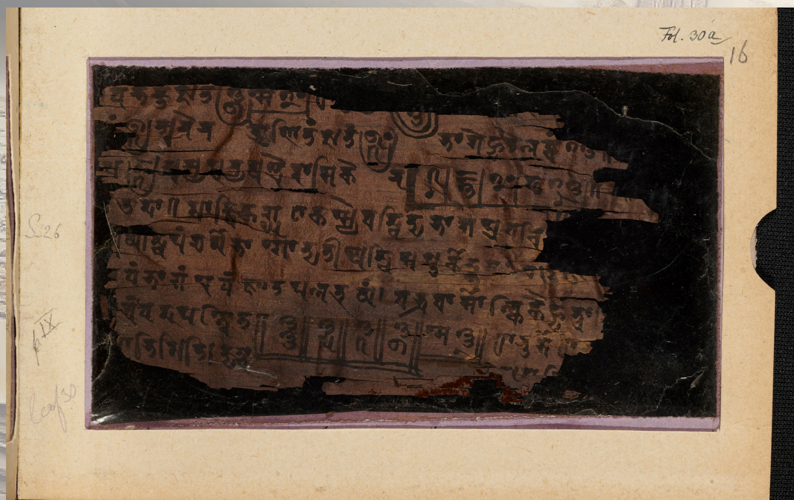
The manuscript is a comprehensive treatise of mathematical principles and illustrations, encompassing subjects such as arithmetic, algebra, geometry, and measurement. It is composed in a variant of literary Sanskrit, infused with regional dialectical influences, and employs numerals with a decimal system, incorporating a dot notation for zero [1]. Some distinguishing characteristics of the manuscript include:

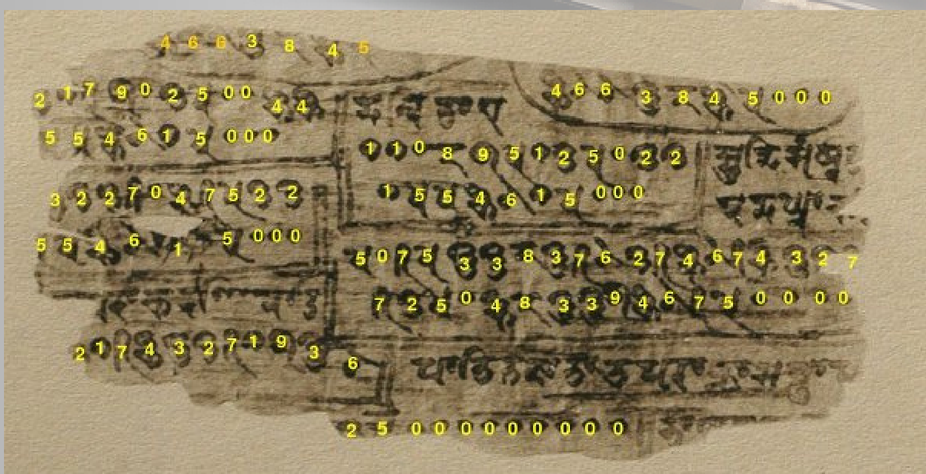
- Mathematical methodologies: Solutions to linear and quadratic equations, arithmetic and geometric sequences, and indeterminate equations of the second order.
- Zero notation: The manuscript utilizes a dot symbol to represent zero, marking one of the earliest recorded instances of zero in Indian mathematical traditions.
- Methodical approach: The manuscript adheres to a systematic methodology for problem-solving, presenting statements of principles, illustrative examples, and operational demonstrations.

Images of Bakhshali Manuscript









Inca's Quipu: A Historical Wonder

-Saswati Ghosh

The Inca Quipu! A fascinating and enigmatic ancient artifact that has captivated historians, mathematicians, and archaeologists for centuries. Let's unravel the mysteries of the Quipu:

 # What is a Quipu?

A Quipu (khipu) is an ancient Inca device used for recording numerical data, keeping track of time, and storing information. It consists of a primary cord with multiple secondary cords, each with a specific number of knots.

 # History of Quipu

The Quipu (khipu) is an ancient device used by the Incas to record numerical data, keep track of time, and store information. The origins of the Quipu date back to the time of the ancient Nazca culture (200 BCE - 600 CE). However, it was during the Inca Empire (1438-1533 CE) that the Quipu became a widespread and sophisticated tool.

 # Construction of Quipu

A Quipu consists of:

1. Primary cord: A main cord made of cotton or wool, which serves as the base of the Quipu.
2. Secondary cords: Multiple cords attached to the primary cord, each with a specific number of knots.
3. Knots: Various types of knots, including simple knots, figure-eight knots, and wrapped knots, which represent different numerical values.

 # Types of Quipu

There are several types of Quipu:

1. Simple Quipu: Used for basic numerical record-keeping.
2. Calendar Quipu: Used to track time and record important dates.
3. Narrative Quipu: Used to record stories, myths, and legends.
4. Accounting Quipu: Used for administrative and economic record-keeping.

Mathematical Significance of Quipu
Quipu demonstrate a sophisticated understanding of mathematics:

1. Base-10 system: Quipu use a base-10 system, with knots representing units, tens, hundreds, and thousands.
2. Positional notation: The placement of knots on the secondary cords indicates their value, similar to positional notation in modern mathematics.
3. Zero concept: The absence of a knot in a specific position may have represented zero.

Deciphering Quipu
Despite their importance, Quipu remain partially deciphered:

1. Limited understanding: The exact meaning and interpretation of Quipu are still not fully understood.
2. Lack of written records: The Incas left no written records explaining the use and meaning of Quipu.
3. Ongoing research: Scholars continue to study Quipu, using mathematical, historical, and anthropological approaches.

Examples of Quipu
Some notable examples of Quipu include:

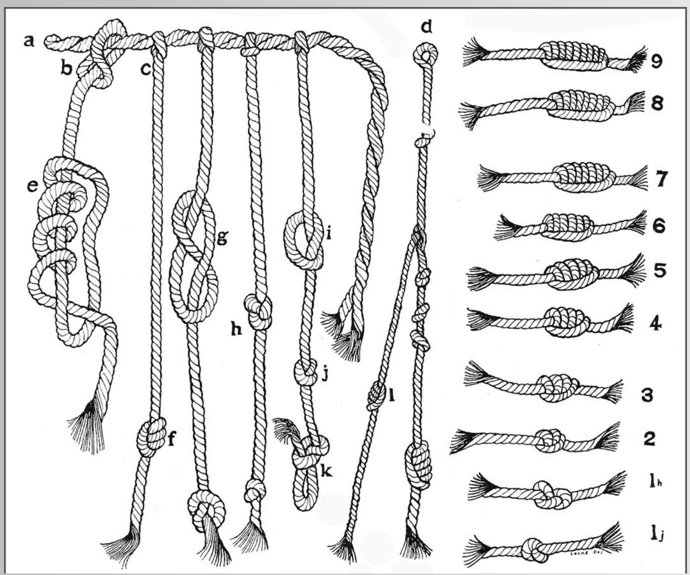
1. The Puruchuco Quipu: Discovered in 1966, this Quipu is one of the most well-preserved and extensively studied.
2. The Inca Quipu Museum: Located in Cusco, Peru, this museum houses a collection of Quipu and offers insights into their construction and use.

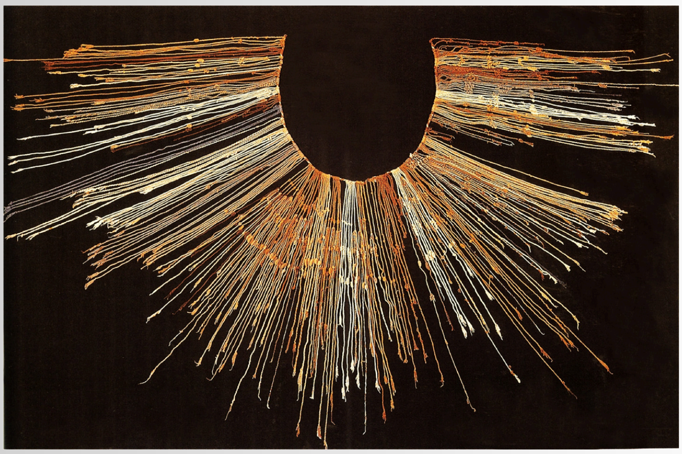
Preservation and Conservation of Quipu
Efforts are being made to preserve and conserve Quipu:

1. Museum collections: Many Quipu are housed in museums, where they are protected and conserved.
2. Digital preservation: Digital images and 3D scans of Quipu are being created to preserve their structure and content.
3. Community engagement: Local communities are being involved in the preservation and conservation of Quipu, ensuring their cultural significance is respected.

The Inca Quipu is an remarkable example of ancient ingenuity and mathematical sophistication. Ongoing research and deciphering efforts will hopefully uncover more secrets of this enigmatic device, providing a deeper understanding of Inca culture and mathematics.

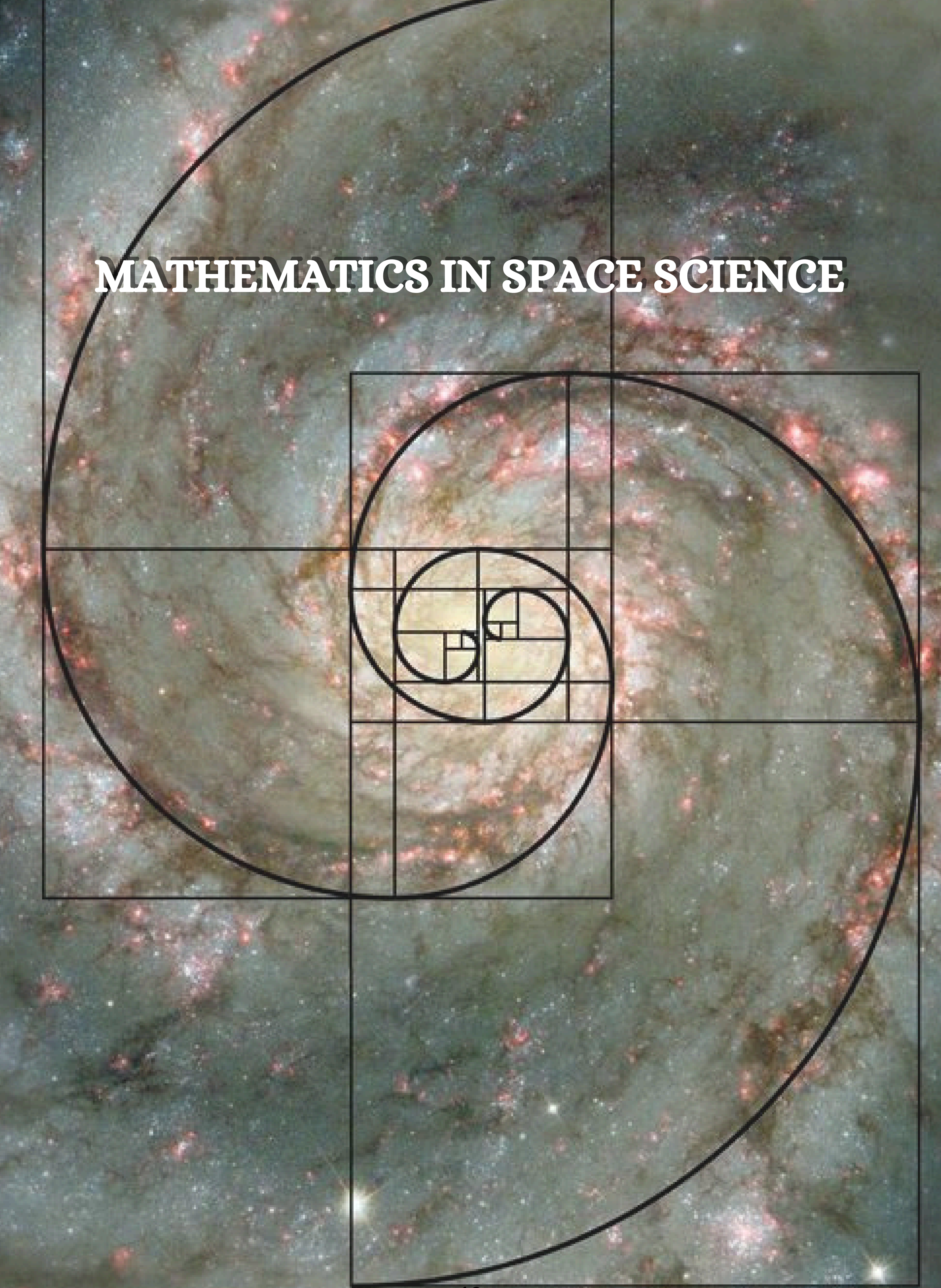
Images of Quipu





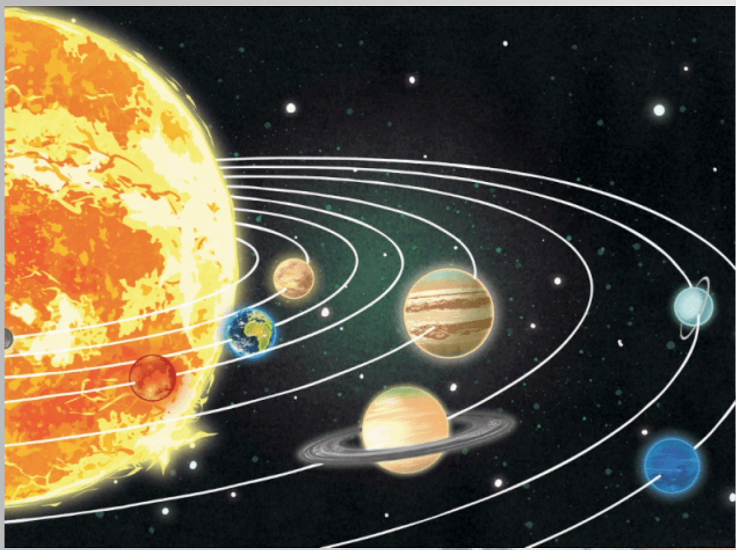
MATHEMATICS IN DAILY LIFE

MATHEMATICS IN SPACE SCIENCE



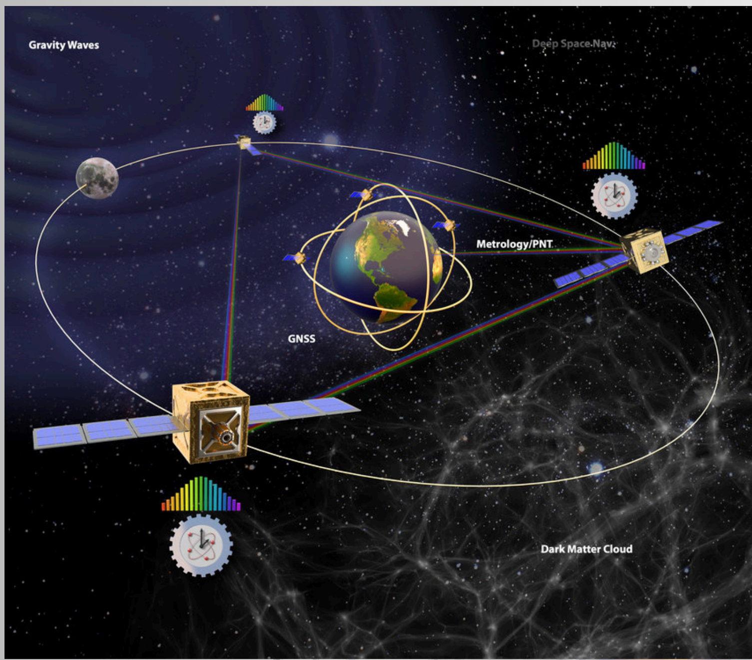
Mathematics is a fundamental tool in space science, enabling researchers to understand and analyze complex phenomena in the universe. Mathematical concepts and techniques are used to describe the motion of celestial bodies, model the behavior of complex systems, and analyze large datasets from space missions. From orbit mechanics to cosmology, mathematics plays a crucial role in advancing our knowledge of the universe.

ORBIT MECHANICS



Mathematics plays a crucial role in orbit mechanics, which is the study of the motion of celestial objects and spacecraft. Mathematical concepts such as Kepler's laws, Newton's law of universal gravitation, and differential equations are used to predict the orbits of objects in space. Additionally, mathematical tools like numerical analysis and computational methods are employed to simulate and analyze orbital mechanics problems. By applying mathematical concepts and techniques, space agencies and researchers can design and optimize spacecraft trajectories, predict orbital paths, and ensure the success of space missions.

NAVIGATION



Mathematics is essential for navigation in space science, enabling spacecraft to accurately determine their position, velocity, and trajectory. Mathematical concepts such as trigonometry, geometry, and calculus are used to calculate navigation parameters like distance, speed, and direction. Additionally, mathematical algorithms and techniques like Kalman filtering and orbital mechanics are employed to process navigation data from sources like GPS, star trackers, and inertial measurement units.

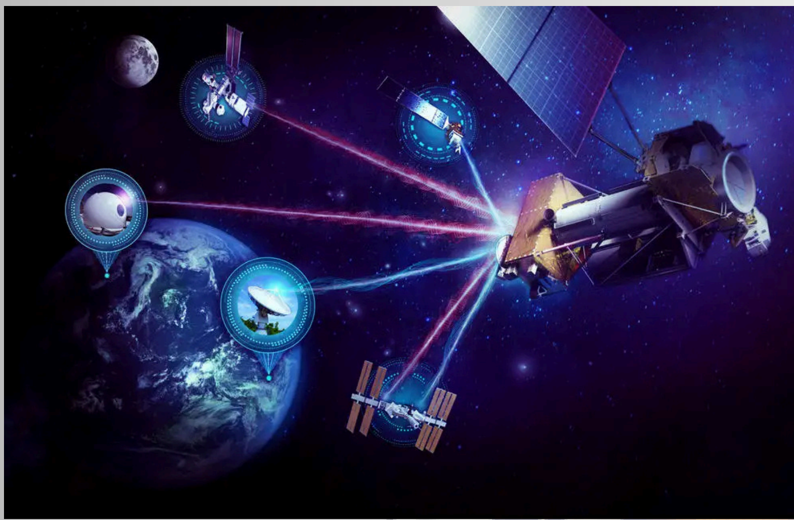
DESIGNING SPACECRAFTS

Mathematics plays a vital role in designing and building spacecraft. Mathematical concepts like algebra, geometry, and calculus are used to optimize spacecraft shape, size, and structure. Mathematicians and engineers employ techniques like finite element analysis, computational fluid dynamics, and optimization algorithms to ensure spacecraft can withstand extreme temperatures, radiation, and mechanical stress.

EXOPLANET DISCOVERY

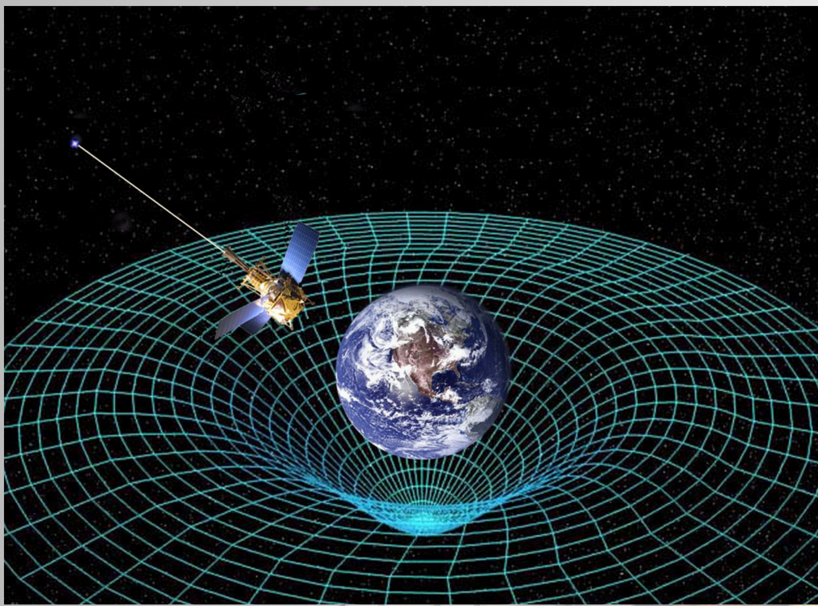
Mathematical techniques like statistical analysis, signal processing, and machine learning are used to analyze data from telescopes and space missions. Specifically, mathematicians employ algorithms to identify patterns in stellar light curves, radial velocity measurements, and transit photometry, helping to confirm the presence of exoplanets and determine their properties, such as size, mass, and orbit.

COMMUNICATION



Mathematics plays a key role in space communication, enabling reliable data transmission between spacecraft and Earth. Mathematical concepts like information theory, coding theory, and signal processing are used to develop error-correcting codes, modulate signals, and optimize data transmission protocols. Additionally, mathematicians employ techniques like Fourier analysis and wavelet analysis to compress and reconstruct signals, ensuring efficient and accurate communication in space.

GENERAL RELATIVITY



Mathematics is central to General Relativity, Albert Einstein's theory of gravity. Mathematical concepts like differential geometry, tensor analysis, and Riemannian geometry are used to describe curved spacetime, gravitational fields, and the motion of massive objects. The Einstein Field Equations, a set of 10 nonlinear partial differential equations, form the mathematical core of General Relativity.

Alumni's Corner

Fermat's Last Theorem : A History of 300 Years — **-Dipanjana Maity**

The Mathematics of Privacy: A Dive into SMPC Protocols and its Real World Applications — **-Sohan Das**

The Mathematics of Privacy: A Dive into SMPC Protocols and its Real World Applications

Sohan Das

In today's world, privacy and security have become paramount, especially when dealing with sensitive information. Secure Multi-party Computation (SMPC) offers a novel cryptographic approach to achieving collective computations without revealing individual data.

Let's explore this fascinating concept through some simple scenarios as follows :

1. Salary Sharing Without Compromising Privacy :

Consider three mutually distrusting colleagues, Alice, Bob, and Jenny, each possessing confidential data—in this case, their salaries. They wish to compute the average of their salaries. However, none of them is willing to disclose their individual salary amount to others. How can they achieve the average salary while maintaining their privacy? The answer lies in SMPC, which enables secure data sharing and computation.

Parties / Entities	Salaries (Confidential)	Shares Generation	Shares Distribution
Alice	$Sal_A = 40K$	$A_1 = 30K, A_2 = 25K, A_3 = -15K$	$D_A = (A_1 + B_2 + J_3)$
Bob	$Sal_B = 50K$	$B_1 = 12K, B_2 = -7K, B_3 = 45K$	$D_B = (B_1 + J_2 + A_3)$
Jenny	$Sal_J = 60K$	$J_1 = 15K, J_2 = 55K, J_3 = -10K$	$D_J = (J_1 + A_2 + B_3)$

Generate Shares:

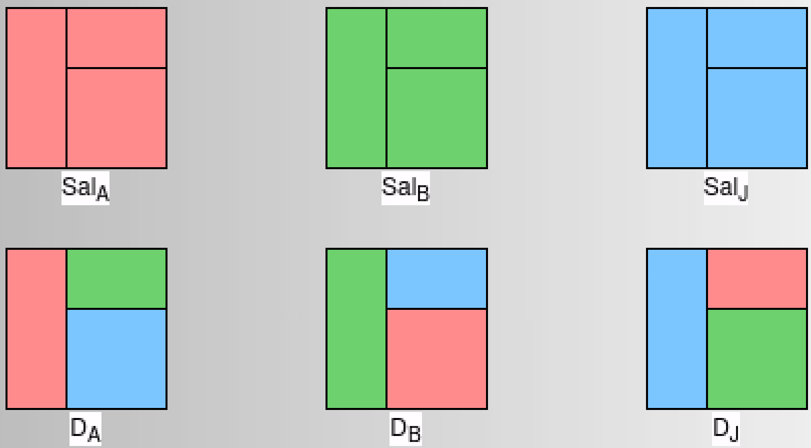
Each individual generates random shares of their salary that sum up to their actual salary. For example, Alice's salary, denoted as Sal_A, is $40K$. She generates her three shares of her salary as follows:

$$A_1 \leftarrow rand(.)\,, A_2 \leftarrow rand(.) \ and \ A_3 = (\,Sal_A - A_1 - A_2\,)$$

Bob and Jenny generate their salary shares in the similar way.

Distribute Shares:

The generated shares are distributed among the entities such that no single party has all the shares of any individual. Each entity computes their distributed shares as shown in the above table.



Compute the Average

The entities collectively compute the average function using their distributed shares.

$$avg(.) = f(x, y, z) \ = (x + y + z)/3$$

$$avg(Salaries) = f(Sal_A, Sal_B, Sal_J) = (40K + 50K + 60K)/3 = 50K$$

$$avg(Shares) = f(D_A, D_B, D_J) = (13K + 52K + 85K)/3 = 50K$$

Conclusion

The security of SMPC lies in the randomness and distribution of the shares. Each individual's salary is never exposed. By leveraging cryptographic protocols, the participants ensure that no one can infer any other's salary from the distributed shares. This example of salary sharing demonstrates the core principle of SMPC.

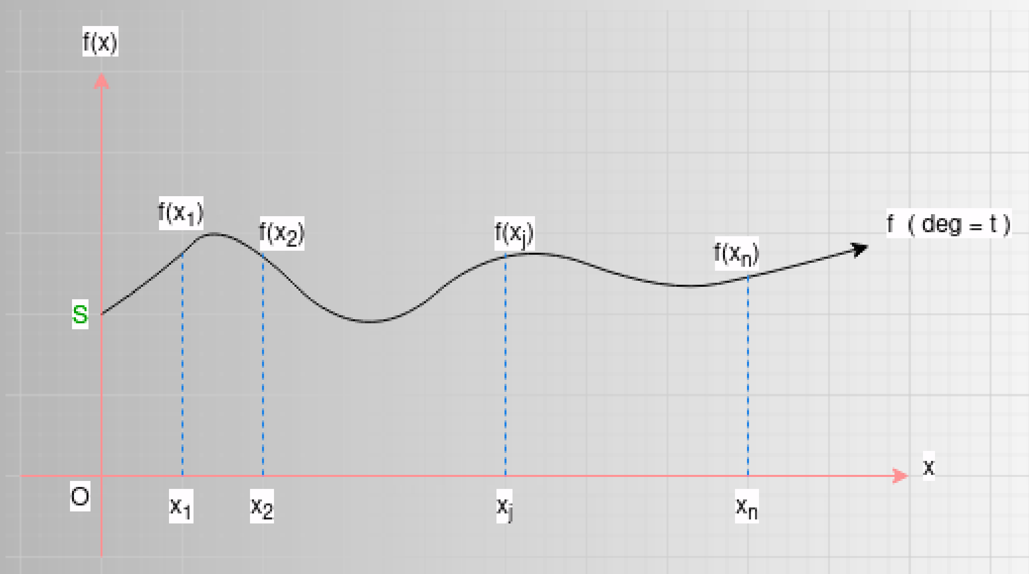
2. Shamir's Secret Sharing (SSS)[1]

Consider that a nuclear arsenal has a secret launch code. The launch code needs to be secured to prevent unauthorised access, ensuring availability when required. Shamir Secret Sharing can be utilised in this situation; the launch code will be split into shares, and these shares will be distributed among n high-ranking officials. Only a predefined threshold , t number of officials can reconstruct the code, no fewer authorities than the threshold t can access it.

Shamir's secret sharing is an ideal and perfect (t, n)-threshold scheme based on polynomial interpolation over finite fields. (t, n)-threshold secret sharing scheme means, it's a scheme where n shares are generated from the secret value in such a way that any collection of t number or less than that of shareholders should not learn anything about the secret, whereas $t + 1$ or more than that number of shareholders have a mechanism to recreate the secret uniquely.

Construction

Let $(F, +, .)$ be a finite field and $|F| > n$. Let's consider $s \in F$ be the secret value. Construct a polynomial $f \in F[X] : f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_t x^t$ $, where\ a_i \leftarrow rand_F(.),\ \forall i \in \{1, ..., n\}, a_0 = s \in F\ , and\ obviously\ t < n.$



[1] https://dl.acm.org/doi/pdf/10.1145/359168.359176

Generate and Distribute the Shares

Compute $D_i = (x_i, f(x_i)), s.t. x_i(\neq 0) \in F \;\; \forall i \in \{1,2,\ldots,n\}$, and distribute those shares to n parties in a secure way. So the i^{th} party P_i has the share $D_i = (x_i\, , f(x_i)), \forall i \in \{1,2,\ldots,n\}$.

Verification

To verify that the secret value $s(= a0)$ can be reconstructed if and only if $t + 1$ or more parties provide their shares, we rely on **Lagrange Interpolation Theorem over the finite field.**

Reconstruction Using Lagrange Interpolation

If $t + 1$ or more shares $D_i = (x_i\, , f(x_i))$ are provided, the polynomial $f(x)$ of degree t can be uniquely determined, and the constant term a_0, which is the secret s , can be recovered as follows :

The formula for Lagrange interpolation is:
$f(x) = \sum_{j=1}^{t+1} \quad f(x_j)L_j(x)\, , where\; L_j(x) = \prod_{\{1\leq m\leq t+1, m\neq j\}}(x - x_m)/(x_j - x_m)\, , in\; F.$
To recover $a_0(= s)$, evaluate $f(0) \; = \; \sum_{j=0}^{t+1} \quad f(x_j)L_j(0)$.

Note: For the above calculation, the first $t + 1$ shares $D_i \; \forall i \in \{1,2,\ldots,t + 1\}$are considered for reconstruction. However, the process will work for any collection of $t + 1$ or more shares, regardless of their order or selection, as long as the shares are distinct and valid.

Security measurement with Fewer than $t + 1$ Shares

If fewer than $t + 1$ shares are available, there are infinitely many polynomials of degree t that pass through the given points. As a result, no information about the secret $s\; (= a_0)$ can be inferred. This property ensures the security of the scheme.

Conclusion

The secret s can only be reconstructed if at least $t + 1$ shares are provided, ensuring both the correctness of reconstruction and the security of the secret.

Fermat's Last Theorem :
A History of 300 Years

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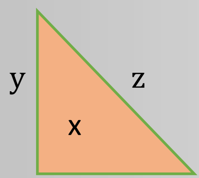
UG: Maulana Azad College (2019-22)

It was 1963 and a ten years old boy Andrew Wiles loved to solve mathematical problems. One day, returning from school, Andrew visited a local library and found a book *'The Last problem'* by Eric Temple Bell. There were so many problems in Mathematics, but the simplicity of Fermat's Last Theorem attracted Wiles to think about it.

From Pythagoras' Theorem to Fermat's Last Theorem:

We all read the famous Pythagoras's Theorem in our childhood, which says that "*In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides*".



i.e., $x^2 + y^2 = z^2$ ---- (*)

The obvious integer solution of (*) is x=3, y=4, z=5. Now, Pythagoreans found many other triples (5,12,13),(99,4900,4901) etc. But as the right triangle becomes larger, finding integer solutions of (*) become tougher. However, Euclid proved that there are infinite number of integer solution of (*).

Andrew knew about the Pythagoras's Theorem and Pythagoreans' Triples from the book *'The Last problem'*, then he saw the equation $\mathbf{x^n + y^n = z^n}, \boldsymbol{n} > 2$, and knew about the conjecture that 'it has no integer solution'. Then he read how over 300 years, many mathematicians had tried to prove this theorem or tried to disprove it and failed. From this library of Milton road, ten years old Andrew dreamed and after 30 years, he proved it, surprising the mathematicians of whole world.

Pierre de Fermat, A Great Mathematician:

In seventeenth century, Mathematics was not a highly regarded subject as today. Pierre de Fermat was a councilor at the chamber of petitions in France. At a time Fermat was very much affected by the book '***Arithmetica'*** by Diophantus, which contains more than one hundred problems and solutions were also given by Diophantus. It was our fortune that there were little margins in each page of *Arithmetica.* Fermat wrote his comments, logics in there and those marginal notes changed the history of mathematics.

One of Fermat's discoveries is related to **'Friendly Numbers'** or 'Amicable Numbers'. Amicable Numbers are pair of numbers such that each number is the sum of the proper divisors of the other number. Pythagoreans first discovered the Amicable Numbers **220 and 284**. After that, at 1636, Fermat discovered the second tuple of Friendly number, **17296 and 18416**. He also noticed that 26 is a number between 25 and 27 where 25 is a square and 27 is a cube. Then he proved that 26 is the unique number between a square and a cube.

At the time of reading the *Book II of Arithmtica*, Fermat looked about the Pythagoras' theorem, Pythagoreans' triples and so many problems about that. At 1637, changing the power from 2 to 3 and more than 3, he wrote at the margin of the book, after problem VIII :

cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos,et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi hanc marginis exiguitas non caperet.

It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have a truly marvelous proof of this, which this margin is too narrow to contain.

Another important work of Fermat is related to prime numbers. Fermat said that *every prime number of the form 4n+1 can be written as sum of two squares and prime numbers of the form 4n-1 can never be written in this way.*[We know that all prime numbers (except 2) are of the form 4n $\pm$ 1]. For example, $\mathbf{13 = 3 \times 4 + 1 = 2^2 + 3^2}$. Later Euler proved this theorem by infinite descent method at 1749.

All the theorems and observations given by Fermat were proved one by one, But the last one was :***There is no integer solution of the equation*** $\boldsymbol{x^n + y^n = z^n, n} > 2$; that's why it is called ***Fermat's Last Theorem***.

Towards the proof of The Fermat's Last Theorem:

Fermat described a sketch of proof of the theorem for n = 4 elsewhere in the book of Arithmetica. Mathematician **Euler** tried to prove the theorem inspiring by the sketch of proof given by Fermat. Euler used the concept of imaginary numbersand gave a prove of the theorem for n = 3. But the proof Euler gave for n = 3 was found to be incomplete and it also can't be generalized for arbitrary n. But Euler's work to prove the theorem was obviously a great development of Mathematics.

If Fermat's Last Theorem could be proved for all primes, then proof was done for any arbitrary n. So, Mathematicians tried to prove the theorem only for prime numbers.

Mathematician **Marie-Sophie Germain** tried to prove the theorem for a special type of primes.Her work to prove the theorem is a great contribution to mathematics. In 1847, **Gabriel Lamè** sketched a proof of the Fermat's last Theorem by complex numbers, cyclotomic field on nth roots of unity . But the gap of his proof was pointed out by Joseph Liouville, who read a paper of Ernst Kummer, which described the failure of unique factorization of complex numbers.

In the mid of 19th century, **Ernst Kummer** proved the Fermat's last Theorem for all Regular primes. Regular prime was defined by Kummer in 1850. An odd prime number p is called regular if it does not divide the class number of the pth cyclotomic field $\mathbb{Q}(\xi_p)$, where ξ_p is the primitive pth roots of unity. The class number of the cyclotomic field is the number of ideals of the ring $\mathbb{Z}(\xi_p)$ up to equivalence. Two ideals I,J are said to be equivalent if there is a non zero u in $\mathbb{Q}(\xi_p)$ such that I=uJ. Kummer successfully filled the lack of unique factorization using cyclotomic field and developing the concept of Ideal numbers.

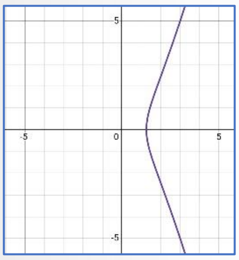
In 1931 and onwards, Gödel's Incompleteness Theorem and Cohen's work confused mathematicians that, 'Is Fermat's Last Theorem provable or unprovable!'

Gödel's First Incompleteness Theorem says that "*If a formal system F (Axiomatic set theory) is consistent, There are statements of the language of F, which can neither be proved nor disproved in F*" . Ans his **Second Incompleteness Theorem** says that " *The consistency of a formal system F can never be provable*".

Andrew Wiles and his effort:

Let's come to the story of Andrew Wiles. In 1975, the PhD guide John Coates introduced Wiles to elliptic curves and the recent works done by mathematicians on it. Elliptic curve is not related to ellipse, it is a plane curve with equation of the form $y^2 = x^3 + ax + b, a, b \in \mathbb{R}$.

The proof of 'The elliptic curve $y^2 = x^3 - 2$ has only one integer solution' was done by Fermat, when he proved that 26 is the only integer sandwiched between a square and a cube.



The elliptic curve $y^2 = x^3 - 2$

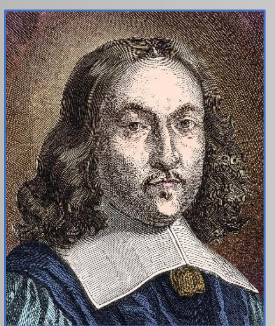
In 1954-55, Japanese mathematicians Goro Shimura and Yutaka Taniyama discovered that there may be a link between elliptic curves and modular forms, two completely different area of

> <u>Taniyama-Shimura Conjecture:</u> Elliptic curve over $\mathbb{Q}$can be obtained via a Rational map with integer coefficient from the classical modular curve $X_0(N)$, for some integer N.

mathematics, which is known as **Taniyama – Shimura Conjecture**.

Then in 1984-85, Mathematicians Frey, Serre and Ribet showed that '*if the Taniyama-Shimura Conjecture could be proven for at least the semi-stable class of elliptic curves, a proof of Fermat's last theorem would follow automatically. i.e., any solution which could contradict Fermat's Last Theorem, can also be used to contradict Taniyama-Shimura Conjecture'*.

Knowing about this invention, Andrew wiles found a path to prove the theorem. He tried to prove that *Taniyama-Shimura Conjecture* is true, using the work of Galöis about the Galöis group. Finally Wiles successfully proved the first step. Later inspired by Kolvagin-Flach's work, Wiles tried to complete the proof. He used hard algebraic geometry and number theory to complete the proof. After seven years of invincible efforts of Wiles, on 21st June, 1993, at Isaac Newton Institute, there was a workshop on mathematics and the title of Wiles' lecture was 'Modular forms, Elliptic curves and Galöis representation'. On 23rd June, the third day of his lecture, mathematicians realized what would happen finally. Andrew Wiles completed the proof and wrote the statement of the Fermat's Last Theorem and said, 'I think I'll stop here' and the 300 years old unsolved mystery was finally solved. At 2016, Andrew Wiles was awarded by the Abel Prize.



Pierre de Fermat

Andrew Wiles

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